Large Scale $B-$Field in Stationary Accretion Disks

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Abstract

We reconsider the problem of the formation of a large-scale magnetic field in the accretion disks around black holes. In contrast with previous work we take into account the nonuniform vertical structure of the disk. The high electrical conductivity of the outer layers of the disk prevents the outward diffusion of the magnetic field. This implies a stationary state with a strong magnetic field in the inner parts of the accretion disk close to the black hole.

keywords: accretion, accretion disks — magnetohydrodynamics — black hole physics

1 Introduction

Early work on disk accretion to a black hole argued that a large-scale magnetic field of, for example, the interstellar medium would be dragged inward and greatly compressed by the accreting plasma (Bisnovatyi-Kogan & Ruzmaikin 1974, 1976; Lovelace 1976). Subsequently, analytic models of the field advection and diffusion in a turbulent disk suggested, that the large-scale field diffuses outward rapidly (Lubow, Papaloizou, & Pringle 1994; Lovelace, Romanova, & Newman 1994) and prevents a significant amplification of the external poloidal field by electrical current in the accretion disk. This has led to the suggestion that special conditions (nonaxisymmetry) are required for the field to be advected inward (Spruit & Uzdensky 2005).

We reconsider the question of the advection/diffusion of a large-scale magnetic field in a turbulent plasma accretion disk, taking into account its nonuniform vertical structure. The high electrical conductivity of the surface layers of the disk, where the turbulence is suppressed by the radiation flux and the relatively high magnetic field, prevents outward diffusion of the magnetic field. This leads in general to a strong magnetic field in the inner parts of accretion disks around black holes.

2 The fully turbulent model

There are two limiting accretion disk models which have analytic solutions for a large-scale magnetic field structure. The first was constructed by Bisnovatyi-Kogan and Ruzmaikin (1976) for a stationary non-rotating accretion disk. A stationary state in this disk (with a constant mass flux onto a black hole) is maintained by the balance between magnetic and gravitational forces, and thermal balance (local) is

maintained by Ohmic heating and radiative radiative conductivity for an optically thick conditions. The mass flux to the black hole in the accretion disk is determined by the finite conductivity of the disk matter and the diffusion of matter across the large-scale magnetic field as sketched in Figure 1. The value of the large-scale magnetic field in stationary conditions is determined by the accretion disk mass, which in turn is determined by the magnetic diffusivity of the matter. For a laminar disk with Coulomb conductivity (which is very large), the mass of the stationary disk is also very large making the disk's self-gravity important. Correspondingly, the magnetic field needed to support a mechanical equilibrium is also very large, reaching in the central parts of the disk $\sim 10^{11}$ G for a stellar mass black hole, for a temperature and density at infinity $T_\infty \sim 10^4$ K and for $\rho_\infty \sim 10^{-24}$ g/cm^3. The stationary magnetic field increases with the black hole mass as $\sim M_{bh}^{3/2}$, and with a mass flux $\sim \dot{M}^{3/2}$, where $\dot{M} \sim \rho_\infty T_\infty^{-3/2}$ (Bisnovatyi-Kogan and Ruzmaikin 1976).
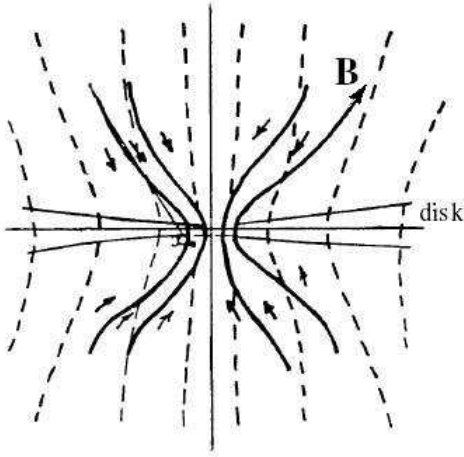

Figure 1: Sketch of the poloidal magnetic field threading an accretion disk (from Bisnovatyi-Kogan & Ruzmaikin 1976). The field strength increases with decreasing radius owing to flux freezing in the accreting disk matter.

It is widely accepted that the laminar disk is un-stable to different hydrodynamic, magnetohydrodynamic, and plasma instabilities which implies that the disk is turbulent. In X-ray binary systems the assumption about turbulent accretion disk is necessary for construction of a realistic models (Shakura and Sunyaev 1973). Therefore the turbulent accretion disks had been constructed also for non-rotating models with a large-scale magnetic field. A turbulent magnetic diffusivity was considered by Parker (1971), and by Bisnovatyi-Kogan and Ruzmaikin (1976). In the last paper the turbulent diffusivity was scaled by the parameters of the turbulent motion, similar to the scaling of the shear α-viscosity in turbulent accretion disk in binaries (Shakura & Sunyaev, 1973) where the viscous stress tensor component $t_{r\phi} = \alpha P$, with $\alpha \le 1$ a dimensionless constant and P the pressure in the disk midplane. In a more consistent representation, the coefficient of turbulent kinematic viscosity ν in the Navier-Stokes equation is taken in the form $\nu = (2/3)\alpha v_s h$, where $v_s = \sqrt{P/\rho}$ is the isothermal sound speed and ρ is the midplane density of the disk. Using this representation, the expression for the turbulent electrical conductivity σ_t is

$$\sigma_t = \frac{c^2}{\tilde{\alpha} 4\pi h \sqrt{P/\rho}}. \qquad (1)$$

Here, $\tilde{\alpha} = \alpha_1 \alpha_2$. The characteristic turbulence scale is $\ell = \alpha_1 h$, where h is the half-thickness of the disk, the characteristic turbulent velocity is $v_t = \alpha_2 \sqrt{P/\rho}$. The mass of the turbulent magnetized disk is orders of magnitude less that in the laminar disk case.

The evolution of a large-scale magnetic field threading a turbulent Keplerian disk can be estimated easily. This field arises from two sources: external electrical currents and currents in the accretion disk. Evidently, the field generated by the currents in the disk can be much larger than that due to the external currents. The magnetic field may become dynamically important, influencing the accretion disk structure and leading to powerful jet formation, only if it is strongly amplified during the radial inflow of the disk matter. This amplification is possible only when the radial accretion speed of matter in the disk is larger than the outward diffusion speed of the poloidal magnetic field due to the turbulent diffusivity $\eta_t = c^2/(4\pi\sigma_t)$. Estimates by Lubow et al.

(1994) shown that for a turbulent conductivity (1), the outward diffusion speed is larger than the accretion speed. Thus it appears that there is no large-scale magnetic field amplification during Keplerian disk accretion. This conclusion is discouraging because the most plausible models of many phenomena observed in the the systems with black holes inside the galaxy, as well as in the extragalactic supermassive black holes are connected with a large values of a large-scale magnetic fields. Nevertheless, this result directly follows from the equations of the standard disk structure, with the turbulent electric conductivity (1)

Lubow et al. (1994) did numerical calculations for a simplified situation with constant relative disk thickness h/r, constant kinematic viscosity ν, and turbulent conductivity σ_t. It is easy to show that the same result follows analytically for the standard accretion disk structure which can be written as

$$\dot{M} = 4\pi\rho v_r r h \ , \quad h = v_s/\Omega_K \ , \quad v_s = \sqrt{P/\rho} \ ,$$

$$4\pi r^2 h\alpha P = \dot{M}(j - j_{in}) \ , \quad \frac{3}{2}\Omega_K\alpha Ph = \frac{2aT^4c}{3\kappa\rho h} \ , \quad (2)$$

where $v_K = r\Omega_K$ is the Keperian velocity (e.g., Bisnovatyi-Kogan & Lovelace 2001). For regions far from the inner disk boundary, the specific angular momentum $j = rv_K \gg j_{in}$. The characteristic time t_{visc} of the accretion disk matter advection due to the shear viscosity is $t_{visc} = r/v_r$. From the first three relations in equation (2), we obtain

$$t_{visc} = \frac{r}{v_r} = \frac{j}{\alpha v_s^2} \ . \qquad (3)$$

To estimate the time-scale of outward magnetic field diffusion, we use

$$t_{diff} = \frac{r^2}{\eta}\frac{h}{r}\frac{B_z}{B_r} \ , \qquad (4)$$

(Lubow et al. 1994), where B_r and B_z are the large scale field components evaluated at the top surface of the disk. Here the coefficient of the magnetic turbulent diffusivity η is obtained from (1)

$$\eta = \frac{c^2}{4\pi\sigma_t} = \tilde{\alpha}hv_s. \qquad (5)$$

For stationary conditions, the large-scale magnetic field in the accretion disk is determined by the equality $t_{visc} = t_{diff}$ which implies

$$\frac{B_r}{B_z} = \frac{\alpha}{\tilde{\alpha}}\frac{v_s}{v_K} = \frac{\alpha}{\tilde{\alpha}}\frac{h}{r} \ll 1 \ . \qquad (6)$$

In contrast, the coronal poloidal field solutions typically have $B_r/B_z \sim 1$ at the disk surface (Bisnovatyi-Kogan and Blinnikov 1972; Ustyugova et al. 1999), which implies that $t_{diff} \ll t_{visc}$. This inequality indicates that the magnetic field is not amplified during accretion for these physical conditions.

3 Turbulent disk with radiative outer zones

Near the surface of the disk, in the region of low optical depth, the turbulent motion is suppressed by the radiative flux, similar to the suppression of the convection over the photospheres of stars with outer convective zones. The presence of the outer radiative layer does not affect the estimate of the characteristic time t_{visc} of the matter advection in the accretion disk because it is determined by the main turbulent part of the disk. The time of the field diffusion, on the contrary, is significantly changed, because the electrical current is concentrated in the radiative highly conductive regions, which generate the main part of the magnetic field. The structure of the magnetic field with outer radiative layers is shown schematically in Figure 2.

Inside the turbulent disk the electrical current is negligibly small so that the magnetic field there is almost fully vertical, with $B_r \ll B_z$, according to (6). In the outer radiative layer, the field diffusion is very small, so that matter advection is leading to strong magnetic field amplification. The field amplification will last until the magnetic forces in the region over the photosphere become of the order of the gravitational ones, and start to participate in the equilibrium balance. In such conditions the MHD and plasma instabilities are developed, decreasing the effective electrical conductivity. We suppose, that in the stationary state the magnetic forces could support the optically thin regions against gravity. In the
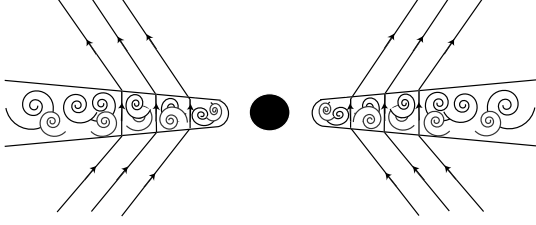
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Figure 2: Sketch of the large-scale poloidal magnetic field threading a rotating turbulent accretion disk with a radiative outer boundary layer. The toroidal current flows mainly in the highly conductive radiative layers. The large-scale (average) field in the turbulent region is almost vertical.

nonrotating magnetized disk magnetic forces support the whole disk against the gravity, so they should be much higher. When the magnetic force balances the gravitational force on the outer optically thin part of the disk of surface density Σ_{ph} one finds the following relation takes place

$$\frac{GM\Sigma_{ph}}{r^2} \simeq \frac{B_z I_\phi}{2c} \simeq \frac{B_z^2}{4\pi} \; , \tag{7}$$

(Bisnovatyi-Kogan & Ruzmaikin 1976). The surface density over the photosphere corresponds to a layer with effective optical depth close to 2/3 (e.g., Bisnovatyi-Kogan 2001). We estimate the lower limit of the magnetic field strength, taking κ_{es} (instead of the effective opacity $\kappa_{eff} = \sqrt{\kappa_{es}\kappa_a}$). Writing

$$\kappa_{es}\Sigma_{ph} = 2/3 \; , \tag{8}$$

we obtain $\Sigma_{ph} = 5/3$ (g/cm^2) for the opacity of the Thomson scattering, $\kappa_{es} = 0.4$ cm^2/g. The absorption opacity κ_a is much less than κ_{es} in the inner regions of a luminous accretion disk. Thus using in equation (7) Σ_{ph} from equation (8), we estimate the lower bound on the large-scale magnetic field of a Keplerian accretion disk as

$$B_z = \sqrt{\frac{5\pi}{3}} \frac{c^2}{\sqrt{GM_\odot}} \frac{1}{x\sqrt{m}} \simeq 10^8 \text{G} \frac{1}{x\sqrt{m}} \; , \tag{9}$$

where $x = r/r_g$ and $m = M/M_\odot$. For comparison, the surface density Σ_d of the disk in the inner radiation dominated region, where we may expect the

largest values of the magnetic fields, is

$$\Sigma_d = \frac{80\sqrt{2}}{9\alpha} \frac{x^{3/2}}{\dot{m}} \left(1 - \sqrt{\frac{3}{x}}\right)^{-1} \; ,$$

$$\dot{m} = \frac{\dot{M}c^2}{L_c} \; , \qquad L_c = \frac{4\pi cGM}{\kappa_{es}} \; , \tag{10}$$

(see Bisnovatyi-Kogan 2001). The maximum magnetic field is reached when the outward magnetic force balances the gravitational force on the disk of surface mass density Σ_{ph}. In equilibrium, $B_z \sim \sqrt{\Sigma_{ph}}$. We find that B_z in a Keplerian accretion disk is about 20 times less than its maximum possible value for $x = 10$, $\alpha = 0.1$, and $\dot{m} = 10$.

4 Discussion

An important question is the energy density of the large-scale stationary magnetic field in comparison with the rotational or gravitational energy density of the disk. For a non-rotating magnetized accretion disk, the energy density of the field is of the order of the gravitational one so that the magnetic field strength is very large in the vicinity of a black hole (Bisnovatyi-Kogan & Ruzmaikin 1976), and may be many order of magnitude larger than the external seed field. In the case of the fully turbulent Keplerian disk the poloidal magnetic field tends to drift outward (Lubow et al. 1994; Lovelace et al. 1994) so that its value cannot significantly exceed the strength of the large-scale seed magnetic field.

The suggestion of a fully turbulent accretion disk with a small turbulent conductivity is violated in the outer surface layer of the accretion disk where the optical depth is small and the turbulence is suppressed by the strong radiative flux. This is similar to the radiative layer above the main body of a convective star (similar to the Sun; e.g., Bisnovatyi-Kogan 2001). In the radiative layer, the magnetic field diffusion is much slower than in the region of fully developed turbulence. In the radiative layer the diffusion is determined by the classical Coulomb conductivity which is very large. The diffusion in this layer is practically negligible. The electric current

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is concentrated in the radiative layer, and the main body of the turbulent disk is almost current-free and thus force-free. The magnetic field lines in this region are almost straight as shown in Figure 2. Because of the negligible diffusion in the radiative layer, the large-scale field drifts inward until the dynamical action of the magnetic field on the photosphere becomes comparable with that of centrifugal and gravitational forces. At this point the inward drift of the field will be halted and a the stationary state formed where the the magnetic, centrifugal, and gravitational forces on the optically thin region will be comparable, and deviations from the Keplerian angular velocity will be significant. The strength of the magnetic field for such conditions is smaller than in the nonrotating disk of Bisnovatyi-Kogan and Ruzmaikin (1976), but still it is very large in the vicinity of the black hole. In this situation we may expect a nonuniform distribution of the angular velocity over the disk thickness: The main body of the turbulent disk is rotates with the velocity close to the Keplerian one, and outer optically thin layers rotate substantially slower.

Self-consistent models of the rotating accretion disks with a large-scale magnetic field requires solution the equations of magnetohydrodynamics. We expect two different self-consistent solutions for the same set of the input parameters. In the case of a fully turbulent disk without radiative surface layers, the large-scale magnetic field will remain close to the value of the seed field and the disk's angular velocity will be close to Keplerian. In the second solution the strength of the magnetic field is large, and it may greatly exceed the strength of the seed field. In this solution the angular velocity distribution may deviate considerably from the Keplerian law. In presence of the radiative layers the solution with a small field will not be stationary, and a transition to the strong field solution will take place. We conclude therefore that the strong field solution is the only stable stationary solution for a rotating accretion disk. Further investigation of the build up of strong large-scale fields by accretion is underway (Rothstein et al. 2007).

Acknowledgements

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